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Reliability Analysis of Slope Stability by Central Point Method

LI Chunge*, WU Congliang**, DUAN Xiaolan***

*(School of civil engineering ＆ architecture, Chongqing Jiaotong University, Chongqing 400074, China)

**(School of Traffic and transportation, Chongqing Jiaotong University, Chongqing 400074, China)

*** (School of civil engineering ＆architecture, Chongqing Jiaotong University, Chongqing 400074, China)

ABSTRACT

Given uncertainty and variability of the slope stability analysis parameter, the paper proceed from the perspective of probability theory and statistics based on the reliability theory. Through the central point method of reliability analysis, performance function about the reliability of slope stability analysis is established. What's more, the central point method and conventional limit equilibrium methods do comparative analysis by calculation example. The approach's numerical results are consistent with the traditional limit equilibrium method and meet the objective reality. The accuracy and practicality of reliability analysis is confirmed in order to provide reliability theory a scientific basis for the feasibility of slope stability analysis.

Keywords – slope stability, uncertainty, reliability analysis, the central point method, limit equilibrium method.

I. Intruduction

Slope is the more common form of the construction about the project. Because of the impact of natural and man-made factors, slope instability generates slump. Therefore, how to design slope and let the disaster caused by slope instability reduce to a minimum is an issue which geotechnical designers need to carefully consider.

At present, the traditional method for slope stability is mainly limit equilibrium method and finite element analysis. The traditional method generally regards the parameters related to the stability as deterministic parameters[1]. However, these certification methods ignore the variability of parameters in time and space. At the same time, a large number of experiments show that the physical and mechanical parameters of rock mass is not definite, but obeys a kind of probability distribution[2]. As the result, Establishing the engineering design and safety assessment on the basis of reliability analysis is new development trend[3]. Based on the reliability theory and slope stability analysis by the central point method, the paper which considers geotechnical variability and uncertainty and combines with calculation examples, compares with limit equilibrium method to confirm the accuracy of the reliability calculation method.

II. Definition of slope reliability analysis and limit state equation

2.1 Definition of slope reliability analysis

Reliability analysis of slope engineering is the method in recent twenty years, which evaluates the safety status of slope. It regards the nature of the slope rock mass, load, groundwater, failure mode, calculation model as random variables. The random variables can be described by combining with some reasonable distribution function. Additionally, the theoretical system of safety state of slope engineering is represented by reliability index or failure probability for drawing lessons from structural engineering reliability theory methods and combining with the specific circumstances of slope engineering. Compared with the traditional deterministic approach, it can better respond to the actual state of slope engineering and solve engineering problems correctly that deterministic methods can not dispose.

If the slope provides with security, durability and other functions, it generally considered to be reliable[4]. According to reliability theory, reliability which refers to the slope is ability to a predetermined function in definite conditions and time, then the value of probability measures whether the function is completed. Based on the equation of state of limit equilibrium theory, slope reliability analysis method developes on the basis of safety valuation factor. Reliability evaluation of slope is still based on a random variable safety factor as a statistical sample, or to characterize the slope status by safety reservation. The fundamental difference is that slope reliability abandons the causal corresponding uncertain relationship of traditional security factors and acknowledges the fact based on pleiotropy. Then reliability is obtained by statistical regularity and the function relation between safety coefficient or margin and the basic variables X_i . Overall, slope reliability analysis is quantification of uncertainty of slope rock mass's steady state, then incorporates these uncertainties into slope designs and analyzes them by the probabilistic method.

2.2 Limit state equation

2.2.1 Stress-strength interference model

The stress-strength interference model is the most useful model in reliability research. Under normal circumstances, the stress and strength are independent variables. The model assumes that if the stress and strength that is allowed have interference, there is the possibility of system failure. Distribution function of stress and strength are f(σ) and g(δ), which are expressed in the same coordinate system, as shown in figure 1.

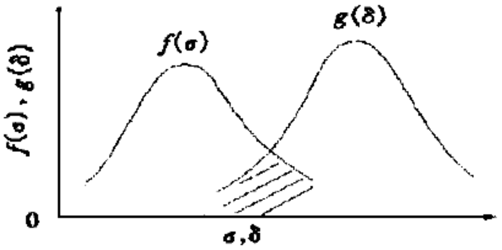

Fig 1. stress-strength interference model

Known from statistics, abscissa of density probability plot of distribution function is the asymptote, then there must be intersecting region, which is the range of structural failure, as shown in the shaded portion in figure 1.

Interference model reveals the nature of probability design. From a statistical view, any design exists the possibility of failure. What the designer can do is control the probability of failure on an acceptable level. There is no probability of failure in the conventional design, therefore, reliability of the design is more reasonable than the conventional design.

2.2.2 Limit state equation

On the assumption of the normal design, construction and use conditions, the slope is no obvious degradation. Therefore, when choosing model, the time-dependent stoc hastic process model is not directly used, but stochastic process model which considers the indirect impact of time is used. Thus, slope reliability depends on the stable state under certain conditions.

Slope stability is controlled by many uncertain factors, such as the failure mechanism, strength and deformation characteristics, groundwater pressure, etc. These uncertainties is random variable. Therefore, performance function which describe the state of slope can be constructed by random variables[5-6]:

$$Z = g(X) = g(X_1, X_2 \dots \dots X_n) \quad (1)$$

Where g(X) is performance function which reflects the function of slope, X is called the basic state variable and these variables are independent each other.

Whether the slope status is damaged depends on safety limit state. Therefore, limit state equation is the next equation:

$$Z = g(X_1, X_2 \dots \dots X_n) = 0 \quad (2)$$

III. The basic principle of central point method

The basic idea of central point method: according to central limit theorem of probability theory, when performance function and basic variable is linear relationship, after performance function linearization, in accordance with Taylor series expansion[7]:

$$Z = g(\mu_{x1}, \mu_{x2}, \dots\dots \mu_{xn}) + \sum_{i=1}^{n} (X_i - \mu_{xi}) \frac{\partial g}{\partial X_i}|_u \quad (3)$$

Statistical parameters about Z:

$$\mu_Z = g(\mu_{x1}, \mu_{x2}, \dots\dots, \mu_{xn}) \quad (4)$$

$$\sigma_z^2 = \sum_{i=1}^{n} \left[(X_i - \mu_{xi}) \frac{\partial g}{\partial X_i}|_u \right] \quad (5)$$

Approximate mean and variance about Z:

$$\mu_Z = g(\mu_{x1}, \mu_{x2}, \dots\dots, \mu_{xn}) \quad (6)$$

$$\sigma_z^2 \approx \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i} \right)^2 |_{\mu_{xi}} \ \sigma_{xi}^2 + \sum_{i \neq j}^{n} \sum^{n} Cov(X_i, X_j) \times \left(\frac{\partial g}{\partial X_i} \right) |_{\mu_{xi}} \left(\frac{\partial g}{\partial X_j} \right) |_{\mu_{xj}} \quad (7)$$

Reliability index is the ratio of the mean and standard deviation of safety margin:

$$\beta = \frac{\mu_z}{\sigma_z} \quad (8)$$

μ_z is the mean value of safety margin and σ_z is the standard deviation of safety margin.

Failure probability:

$$P = 1 - \phi(\beta) \quad (9)$$

The value of P is easy to obtain by checking normal distribution table.1

Tab1. the relationship between β and P

β	1.00	1.64	2.00	3.00	3.09	3.71
P	15.87 × 10^{-2}	5.05 ×10-2	2.27 ×10-2	1.35 × 10^{-3}	1.00 × 10^{-3}	1.04 × 10^{-4}

Reliability:

$$P_k = 1 - P \quad (10)$$

IV. Reliability analysis of slope stability

Factors affecting slope stability is uncertainty, but these uncertainties which have an impact on slope stability can be evaluated by reliability analysis of slope stability[8]. The strength parameter c,φ is a key indicator of influencing slope stability. Prior to the analysis[9], assuming c, φ as random variable of performance function and independent. What's more, c,φ should obey normal distribution.

Select Druker-Prager yield criterion as performance function Z:

$$Z = g(c,\varphi) = I_1 \frac{\sin\varphi}{\sqrt{9+3\sin^2\varphi}} + \sqrt{J_2} - \frac{3c\cos\varphi}{\sqrt{9+3\sin^2\varphi}} \quad (11)$$

I_1, J_2 is function which is unrelated to c,φ:

$$\frac{\partial g}{\partial I_1} = 0, \frac{\partial g}{\partial J_2} = 0 \quad (12)$$

Taylor series expansion about Z in the light of $\mu_{c,}\mu_{\varphi}$:

$$Z = g(c,\varphi)$$

$$Z = g\left(\mu_c,\mu_\varphi\right) + (c-\mu_c)\frac{\partial g}{\partial c} + \left(\varphi-\mu_\varphi\right)\frac{\partial g}{\partial \varphi} + \frac{1}{2}(c-\mu_c) \times \left(\varphi-\mu_\varphi\right) \times \frac{\partial^2 g}{\partial c\,\partial\varphi} \quad (13)$$

First-order approximation the mean and variance of Z:

$$\mu_z = g(c,\varphi)$$

$$\sigma_z^{\ 2} \approx \left(\frac{\partial g}{\partial c}\right)^2 \sigma_c^{\ 2} + \left(\frac{\partial g}{\partial \varphi}\right)^2 \sigma_\varphi^{\ 2} + Cov(c,\varphi)\times\frac{\partial^2 g}{\partial c\partial\varphi} \quad (14)$$

Partial derivative of equation 11:

$$\frac{\partial g}{\partial c} = \frac{I_1\cos\varphi + 3c\sin\varphi}{\sqrt{9+3\sin^2\varphi}} + \frac{6\left(3c\sin\varphi\cos^2\varphi - I_1\sin^2\varphi\cos\varphi\right)}{\left(9+3\sin^2\varphi\right)^{3/2}} \quad (15)$$

$$\frac{\partial g}{\partial \varphi} = \frac{3\cos\varphi}{\sqrt{9+3\sin^2\varphi}} \quad (16)$$

c,φ is independent each other , so , $Cov(c,\varphi) = 0$

$$\sigma_z^{\ 2} \approx \left(\frac{\partial g}{\partial c}\right)^2 \sigma_c^{\ 2} + \left(\frac{\partial g}{\partial \varphi}\right)^2 \sigma_\varphi^{\ 2} \quad (17)$$

According to the above derivation, point reliability calculation of the strength of slope stability can be carried out.

The central point method can directly get the relationship between reliability index and random variable parameter, but it also has the inevitable drawbacks: The performance function expansion in the place of mean value is not entirely reasonable; It is only suitable for variable which is normal distribution or lognormal distribution[10]; State function ignore the secondary or higher order terms. Therefore, for non-linear performance function, there are a lot of errors.

V. Application

The accuracy and usefulness of this method can be verified by the example of soil slope stability in the paper.

Table.1 shows that shear strength index value of rock and soil mass can be obtained by original material and laboratory test.

Reliability index of local points of the slope is calculated by using the statistical value as table 1. Table.2 shows reliability value of slope and toe points.

Tab2. shear strength index value of rock and soil mass

State	C/MPa		φ/°	
	Main	Variance	Main	Variance
Saturated Soil	0.200	0.006	16	1.6

Tab3. reliability value of slope and toe points

Point Number	1	2	3	4	5
Reliability Index，β	0.7768	0.7125	0.6874	0.6235	0.8459
Failure Probability ，P/%	21.8	24.1	24.5	24.9	21.0

Table.2 shows that the reliability of the slope points are low, the reliability index β are less than 1 and the failure probability are more than 20%. The slope safety factor of stability is 1.1752 in point number 1. What shows in table.2 indicates that these points are in danger zone of slope. The data results of this method are consistent with the traditional limit equilibrium method(safety factor of stability is 1.174). That is to say, the results agree with objective facts.

VI. Conclusion

The paper argues that: in order to make a reasonable, proper evaluation for slope stability and provide precise basis for slope design, the reliability theory and method is introduced into slope stability analysis. The reliability method can provide better services engineering practice and achieve the correct evaluation of slope stability with the traditional limit equilibrium method. Reliability design of slope stability has a good foundation, which will be the development trend of slope stability analysis.

Based on the studies and surveys, following conclusion can be obtained :

(1) The calculation results of central point method are basically consistent with the traditional limit equilibrium method, therefore, based on the central point method, research of slope stability is feasible.

(2) Based on traditional valuation method, reliability analysis method is more advanced, however, it doesn’t mean that this method will replace traditional valuation method. Therefore, reliability analysis of slope stability can be introduced for a comprehensive evaluation of the slope.

(3) Safety factor of stability which is calculated is greater than 1 by reliability analysis, but the possibility of instability and failure is 21.8% in point number 1. Therefore, safety factor which is greater than 1 doesn’t mean that the slope is stable and safe.

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